Homework 2 CSE 310 Fall 2008 Arizona State University

Due by Tuesday 9/9 at 9:00

1. Rank the following functions by order of growth; that is, find an arrangement $g_1, g_2, \ldots, g_{15}$ of the functions satisfying $g_1 = \Omega(g_2)$, $g_2 = \Omega(g_3)$, etc. Partition your list into equivalence classes such that $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$.

   $(\sqrt{2})^{\lg n}$, n^2, $n!$, n^3, $\lg^2 n$, $n^{1/\lg n}$, $\ln n$, 1, $2^{\lg n}$, e^n, n, 2^n, $n \lg n$, $\sqrt{\lg n}$, $\ln \ln n$.

2. Show that in any subtree of a max-heap, the root of the subtree contains the largest value occurring anywhere in that subtree.

3. Using Figure 6.3 in the textbook as a model, illustrate the operation of Build-Max-Heap on the array $A = \langle 5, 3, 17, 10, 84, 19, 6, 22, 9 \rangle$.

4. Show that there are at most $\lceil n/2^{h+1} \rceil$ nodes of height h in any n-element heap.

5. What is the running time of heapsort on an array A of length n that is already sorted in increasing order? What about decreasing order?

6. The operation Heap-Delete(A, i) deletes the item in node i from heap A. Give an implementation of Heap-Delete that runs in $O(\lg n)$ time for an n-element heap.