Due by Tuesday 10/28 at 9:00

1. Give a sequence of m Make-Set, Union and Find-Set operations, n of which are Make-Set operations, that takes $\Omega(m \lg n)$ time when we use union by rank but not path compression.

2. Give a simple example of a directed graph with negative-weight edges for which Dijkstra's algorithm produces incorrect answers. Why doesn't the proof of Theorem 24.6 go through when negative-weight edges are allowed?

3. Suppose you are traveling in a very sparsely populated area, and your vehicle has a limited range (uses up fuel very quickly). For example, think of a road network at which some of the vertices are also refueling stations. You know the map perfectly, including all the points at which you can refuel. Denote by D the largest distance you can cover without stopping at a refueling station. Describe an efficient algorithm that finds a shortest path from an origin s to a destination d that will not leave you stranded without fuel. You may describe your algorithm using the algorithms we have studied in class. Give the best expression you can for the running time of your algorithm and explain what data structures it needs to achieve that running time.

4. We are given a directed graph $G = (V, E)$ on which each edge $(u, v) \in E$ has an associated value $r(u, v)$, which is a real number in the range $0 \leqslant r(u, v) \leqslant 1$ that represents the reliability of a communication channel from vertex u to vertex v. We interpret $r(u, v)$ as the probability that the channel from u to v will not fail, and we assume that these probabilities are independent. Give an efficient algorithm to find the most reliable path between two given vertices.