

Due: Thursday, 4/24/2008 before class

1. Let ϕ be a 3cnf-formula. An $\neq$ -assignment to the variables of ϕ is one where each clause contains two literals with unequal truth values. In other words, an $\neq$ -assignment satisfies ϕ without assigning three true literals to any clause.

- (a) Show that the negation of an $\neq$ -assignment is also an $\neq$ -assignment.
- (b) Let $\neq SAT$ be the collection of 3cnf formulas that have an $\neq$ -assignment. Show that we obtain a polynomial-time reduction from $3SAT$ to $\neq SAT$ by replacing each clause c_i

$$(y_1 \vee y_2 \vee y_3)$$

with the two clauses

$$(y_1 \vee y_2 \vee z_i) \wedge (\bar{z}_i \vee y_3 \vee b),$$

where z_i is a new variable for each clause c_i and b is a single additional new variable.

- (c) Conclude that $\neq SAT$ is NP-complete.
2. A **cut** in an undirected graph is a separation of the vertex set V into two disjoint subsets S and T . The size of a cut is the number of edges that have one endpoint in S and the other in T . Let $MAXCUT = \{\langle G, k \rangle \mid G \text{ has a cut of size } k \text{ or more}\}$. Show that $MAXCUT$ is NP-complete. You may assume the result of the previous problem. (Hint: show that $\neq SAT \leq_P MAXCUT$. The variable gadget for variable x is a collection of $3c$ nodes labeled with x and another $3c$ nodes labeled with $\bar{x}$, where c is the number of clauses. All nodes labeled x are connected with all nodes labeled $\bar{x}$. The clause gadget is a triangle of three edges connecting three nodes labeled with the literals appearing in the clause. Do not use the same node in more than one clause gadget. Prove that this reduction works.)
 3. A *coloring* of a graph is an assignment of colors to its nodes so that no two adjacent nodes are assigned the same color. Let $3COLOR = \{\langle G \rangle \mid \text{the nodes of } G \text{ can be colored with three colors so that no two nodes joined by an edge have the same color}\}$. Show that $3COLOR$ is NP-complete. (Hint: use the subgraphs drawn in class to construct your reduction.)